

Casimir energy and the possibility of higher dimensional manipulation

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Abstract

It is well known that the Casimir effect is an excellent candidate for the stabilization of the extra dimensions. It has also been suggested that the Casimir effect in higher dimensions may be the underlying phenomenon that is responsible for the dark energy which is currently driving the accelerated expansion of the universe. In this paper we suggest that, in principle, it may be possible to directly manipulate the size of an extra dimension locally using Standard Model fields in the next generation of particle accelerators. This adjustment of the size of the higher dimension could serve as a technological mechanism to locally adjust the dark energy density and change the local expansion of spacetime. This idea holds tantalizing possibilities in the context of exotic spacecraft propulsion.

1 Introduction

Many of the high energy theories of fundamental physics are formulated in higher dimensional spacetimes. In particular, the idea of extra dimensions has been extensively used in supergravity and superstring theories. It is commonly assumed that the extra dimensions are compactified. From an inflationary point of view, universes with compact spatial dimensions, under certain conditions, should be considered a rule rather than an exception [1]. The models of a compact universe with non-trivial topology may play important roles by providing proper initial conditions for inflation.

The compactification of spatial dimensions leads to a number of interesting quantum field theoretic effects, which include instabilities in interacting field theories, topological mass generation and symmetry breaking. In the case of non-trivial topology, the boundary conditions imposed on fields give rise to the modification of the spectrum for vacuum fluctuations and, as a result, to the Casimir-type contributions in the vacuum expectation values of physical observables.

The Casimir effect is arguably the most poignant demonstration of the reality of the quantum vacuum and can be appreciated most simply in the interaction of a pair of neutral parallel plates. The presence of the plates modifies the quantum vacuum, and this modification causes the plates to be pulled toward each other with a force $F \propto 1/a^4$ where a is the plate separation. The Casimir effect is a purely quantum phenomenon. In classical electrodynamics the force between the plates is zero. The ideal scenario occurs at zero temperature when there are no real photons (only virtual photons) between the plates; thus, it is the ground state of

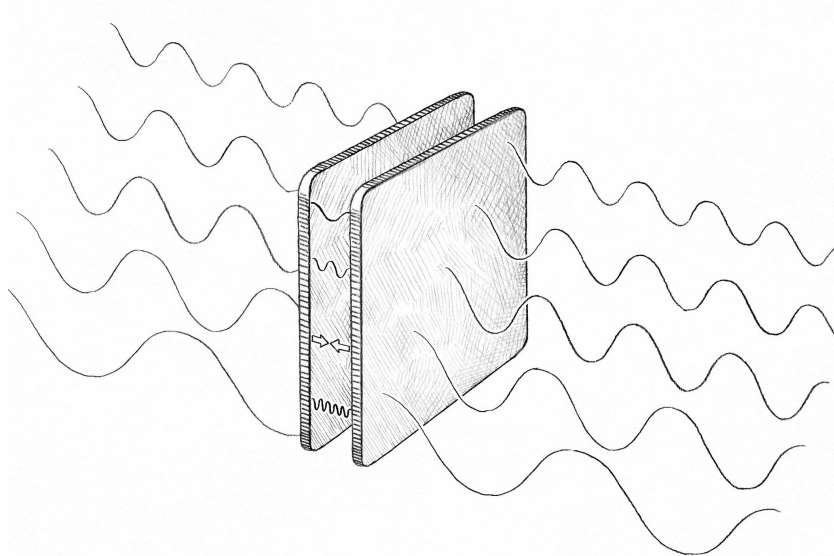


Figure 1: Due to the non-trivial boundary conditions imposed on the quantum vacuum, the plates are pulled toward each other due to a force that is purely quantum in nature.

the quantum electrodynamic vacuum which causes the attraction. One of the most important features of the Casimir effect is that even though it is purely quantum in nature, it manifests itself macroscopically (Figure 1).¹

Compactified extra dimensions introduce non-trivial boundary conditions to the quantum vacuum similar to those in the parallel plate example, and Casimir-type calculations become important when calculating the resulting vacuum energy (for the topological Casimir effect and its role in cosmology see [2] and references therein).

One important question in the context of theories with extra dimensions is why additional spatial dimensions hold some fixed size. This is commonly referred to as ‘modulus stabilization’. Broadly stated, the question is as follows: if there are additional spatial dimensions, why do they not perpetually expand, like our familiar dimensions of space, or alternatively, why do they not perpetually contract? What mechanism is it that allows for this higher space to remain compact and stable? Of the handful of theories that attempt to answer this problem, the Casimir effect is particularly appealing due to its naturalness.

We find this effect compelling due to the fact that it is a natural feature intrinsic to the fabric of space itself. There is much research in the literature that demonstrates that with some combination of fields, it is possible to generate a stable minimum of the potential at some extra dimensional radius [3]- [27]. In addition, the Casimir effect has been used as a stabilization mechanism for moduli fields which parameterize the size and the shape of the extra dimensions both in models with a smooth distribution of matter in the extra dimensions and in models with matter located on branes.

The Casimir energy can also serve as a model for dark energy needed for the explanation of the present accelerated expansion of the universe (see [28]- [30] and references therein). One interesting feature of these models is the strong dependence of the dark energy density on the size of the extra dimension.

One compelling possibility that we would like to introduce in this paper is that at the energies accessible in the next generation of particle accelerators, Standard Model (SM) fields

¹Illustration courtesy of Richard Obousy Consulting LLC and AlVin, Antigravité

might interact with the graviton Kaluza-Klein (KK) tower which would effect the local minimum of the potential. This would have the effect of locally adjusting the radius of the extra dimension during this interaction. Because the dark energy density is a function of the size of the extra dimension, one remarkable feature of this idea is that because the extra dimensions may be (temporarily) adjusted, so too would the local dark energy density. This adjustment would mean that, for the duration of the interaction, the expansion of spacetime would be changed due to technological intervention. What is particularly appealing about this predicted phenomenon is its potential application as a future exotic spacecraft propulsion mechanism.

In the present paper we shortly review the uses of the Casimir effect in both standard Kaluza-Klein type and braneworld scenarios for the stabilization of extra dimensions and for the generation of dark energy. We also explore the energy requirements that would be needed to temporarily adjust the size of the higher dimension and hypothesize that, with some imagination, this mechanism could be used by a sufficiently advanced technology as a means of spacecraft propulsion.

2 Kaluza-Klein type models

To be consistent with the observational data, the extra dimensions in the standard Kaluza-Klein description are assumed to be microscopic, with a size much smaller than the scale of four dimensions. A generic prediction of theories involving extra dimensions is that the gauge and Yukawa couplings are in general related to the size of extra dimensions. In a cosmological context, this implies that all couplings depend on the parameters of the cosmological evolution. In particular, if the corresponding scale factors are dynamical functions their time dependence induces that for the gauge coupling constants. However, the strong cosmological constraints on the variation of gauge couplings coming from the measurements of the quasar absorption lines, the cosmic microwave background, and primordial nucleosynthesis indicate that the extra dimensions must be not only small but also static or nearly static. Consequently, the stabilization of extra dimensions in multidimensional theories near their present day values is a crucial issue and has been investigated in a number of papers. Various mechanisms have been considered including fluxes from form-fields, one-loop quantum effects from compact dimensions, wrapped branes and string corrections.

Let us consider the higher-dimensional action

$$S = \int d^D x \sqrt{|\det g_{MN}|} \left\{ -\frac{1}{16\pi G} R[g_{MN}] + L \right\}, \quad (1)$$

with the matter Lagrangian L which includes also the cosmological constant term. We take a spacetime of the form $R \times M_0 \times \dots \times M_n$ with the corresponding line element

$$ds^2 = g_{MN} dx^M dx^N = g_{\mu\nu} dx^\mu dx^\nu + \sum_{i=1}^n e^{2\beta_i} g_{m_i n_i} dx^{m_i} dx^{n_i}, \quad (2)$$

where M_i , $i = 0, 1, \dots, n$, are d_i -dimensional spaces, $g_{\mu\nu}$, β_i are functions of the coordinates x^μ in the subspace $R \times M_0$ only, and the metric tensor $g_{m_i n_i}$ in the subspace M_i is a function of the coordinates x^{m_i} in this subspace only. In order to present the effective action in the subspace $R \times M_0$ in the standard Einsteinian form we make a conformal transformation of the $(d_0 + 1)$ -dimensional metric:

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \Omega = \exp \left[\sum_{j=1}^n d_j \beta_j / (d_0 + 1) \right]. \quad (3)$$

Dropping the total derivatives the action is presented in the form

$$S = \frac{\prod_j \mu^{(j)}}{16\pi G} \int d^{d_0+1} x \sqrt{|\det \tilde{g}_{\mu\nu}|} \left\{ -R[\tilde{g}_{\mu\nu}] + \sum_{i,j} G_{ij} \tilde{g}^{\mu\nu} \partial_\mu \beta_i \partial_\nu \beta_j - 2U \right\}, \quad (4)$$

where $\mu^{(j)} = \int d^{d_j} x \sqrt{|\det g_{m_j n_j}|}$, $G_{ij} = d_i \delta_{ij} + d_i d_j / (d_0 + 1)$, and

$$U = \frac{\Omega^{-2}}{2 \prod_j \mu^{(j)}} \int d^{D-d_0-1} x \prod_j \sqrt{|\det g_{m_j n_j}|} \left\{ \sum_i e^{-2\beta_i} R[g_{m_i n_i}] - 16\pi GL \right\}. \quad (5)$$

In (5), $R[g_{m_i n_i}]$ is the Ricci scalar for the metric $g_{m_i n_i}$.

In the case of standard cosmological metric $\tilde{g}_{\mu\nu}$ with the scale factor \tilde{a}_0 and the synchronous time coordinate \tilde{t} , the field equations for the set of fields $\beta_i = \beta_i(\tilde{t})$ following from action (4) have the form

$$\sum_{j=1}^n G_{ij} \left(\beta_j'' + d_0 \tilde{\beta}_0' \beta_j' \right) = - \left(\frac{\partial U}{\partial \beta_i} \right)_{\tilde{\beta}_0}, \quad i = 1, 2, \dots, n, \quad (6)$$

where $\tilde{a}_0 = \tilde{a}_0^{(0)} e^{\tilde{\beta}_0}$, $a_j = a_j^{(0)} e^{\beta_j}$, the prime means the derivative with respect to the time coordinate \tilde{t} and the potential U is defined by the formula

$$U = \frac{1}{2\Omega^2} \left(16\pi G\rho + \Lambda_D - \sum_{j=1}^n \lambda_j d_j / a_j^2 \right). \quad (7)$$

In (7) ρ is the matter energy density and Λ_D is the D -dimensional cosmological constant, $\lambda_j = k_j(d_j - 1)$, where $k_j = -1, 0, 1$ for the subspace M_j with negative, zero, and positive curvatures, respectively. In the case $\rho = 0$ for the extrema of potential (7) one has the relations $\Lambda_D = (D - 2)\lambda_i e^{-2\beta_i}$ and $\partial^2 U / \partial \beta_i \partial \beta_j = -2G_{ij}\Lambda_D / (D - 2)$. It follows from here that for $\Lambda_D > 0$ the extremum is a maximum of the potential and is realized for internal spaces with positive curvature. The corresponding effective cosmological constant is positive. For $\Lambda_D < 0$ the extremum is a minimum and is realized for internal spaces with negative curvature. The effective cosmological constant is negative.

There is a number of mechanisms giving contributions to the potential in addition to the cosmological constant and curvature terms. An incomplete list includes fluxes from form-fields, one-loop quantum effects from compact dimensions (Casimir effect), wrapped branes, string corrections (loop and classical). In the case of a single extra space for massless fields at zero temperature the corresponding energy density is of the power-law form

$$\rho = \sigma a_1^{-q}, \quad (8)$$

with a constant σ and q being an integer. The values of the parameter q for various mechanisms are as follows: $q = D$ for the contribution from the Casimir effect due to massless fields, $q = d_1 - 2p$ for fluxes of p -form fields, $q = p - d_0 - 2d_1$ for p -branes wrapping the extra dimensions. The corresponding potential has the form (7) with

$$U = \frac{1}{2\Omega^2} \left(16\pi G\sigma a_1^{-q} + \Lambda_D - \lambda_1 d_1 a_1^{-2} \right). \quad (9)$$

In accordance with Eq. (6), solutions with a static internal space correspond to the extremum of the potential U and the effective cosmological constant is related to the value of the potential at the extremum by the formula $\Lambda_{\text{eff}} = 2\Omega^2 U$.

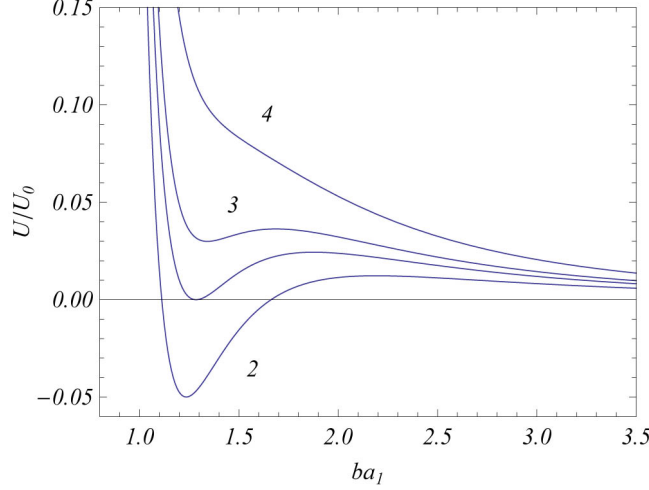


Figure 2: The potential U/U_0 as a function of ba_1 in the model with $d_0 = d_1 = 3$ and with the Casimir effect as a source for ρ . The numbers near the curves correspond to the value of the parameter $\Lambda_D b^2$. For the curve with the zero cosmological constant $\Lambda_D b^2 = (30/7)(2/7)^{2/5}$.

Introducing the notations

$$y = ba_1, \quad b = \left[\frac{d_1(d_1 - 1)}{16\pi G|\sigma|} \right]^{\frac{1}{q-2}}, \quad U_0 = \frac{d_1(d_1 - 1)}{2} b^2 (ba_1^{(0)})^{2d_1/(d_0-1)}, \quad (10)$$

potential (9) is written in the form

$$U = U_0 y^{-q-2d_1/(d_0-1)} \left[\frac{\Lambda_D b^{-2}}{d_1(d_1 - 1)} y^q - k_1 y^{q-2} + \text{sign}(\sigma) \right]. \quad (11)$$

For the extremum with the zero effective cosmological constant one has

$$k_1 y^{q-2} = \frac{q}{2} \text{sign}(\sigma), \quad \Lambda_D b^{-2} = k_1 d_1(d_1 - 1) \left(1 - \frac{2}{q} \right) \left(\frac{2}{q} \right)^{\frac{2}{q-2}}. \quad (12)$$

For positive values q and for an internal space with positive (negative) curvature the extremum exists only when $\sigma > 0$ ($\sigma < 0$). The extremum is a minimum for $k_1(q - 2) > 0$ and a maximum for $k_1(q - 2) < 0$. Potential (11) is a monotonic decreasing positive function for $k_1 = -1, 0$, $\Lambda_D \geq 0$, $\sigma > 0$, has a single maximum with positive effective cosmological constant for $k_1 = 1, 0$, $\Lambda_D > 0$, $\sigma < 0$ and $k_1 = -1$, $\Lambda_D \geq 0$, $\sigma < 0$. For the case $k_1 = 1$, $\Lambda_D > 0$, $\sigma > 0$ and in the $d_0 = d_1 = 3$ model where ρ is generated by one-loop quantum effects ($q = D$ in Eq. (8)) the potential (11) is plotted in Figure 2 for various values of higher dimensional cosmological constant corresponding to $\Lambda_D b^{-2} = 2, (30/7)(2/7)^{2/5}, 3, 4$. In the model with the second value the effective $(d_0 + 1)$ -dimensional cosmological constant vanishes. The behavior of the potential (11) in the other cases is obtained from those described above by replacements $(k_1, \Lambda_D, \sigma) \rightarrow (-k_1, -\Lambda_D, -\sigma)$ and $U \rightarrow -U$.

Introducing the D -dimensional Planck mass M_D in accordance with the relation $G = M_D^{2-D}$, from (10) we see that in the model with one-loop quantum effects the parameter b is of the order of the higher dimensional Planck mass. As it is seen from the graphs in figure 2 the stabilized value of the size for the internal space is of the order of D -dimensional Planck length $1/M_D$. Note that, as it follows from (4), for the effective 4-dimensional Planck mass one has

$M_{\text{Pl}}^2 \approx (a_1 M_D)^{d_1} M_D^2$ and, hence, in this type of models the effective and higher dimensional Planck masses have the same order. Consequently, the size of the internal space is of the order of the Planck length and is not accessible for the near future accelerators. Note that our knowledge of the electroweak and strong forces extends with great precision down to distances of order 10^{-16} cm. Thus if standard model fields propagate in extra dimensions, then they must be compactified at a scale above a few hundred GeV range.

3 Models with large extra dimensions

In the previous section we have considered the models, where the extra dimensions are stabilized by one-loop quantum effects with the combination of curvature terms and the higher dimensional cosmological constant. We have taken the Casimir energy density coming from the compact internal space in the simplest form σa_1^{-D} . The corresponding effective potential is monotonic and cannot stabilize the internal space separately. In more general cases, in particular for massive fields, the dependence of the Casimir energy on the size of the internal space is more complicated and the corresponding effective potential can stabilize the extra dimensions separately.

As a simple model consider the case of a single extra space, $n = 1$, being a circle, $M_1 = S^1$, with a scalar field φ as a non-gravitational source. In the discussion below we will assume that either the external space M_0 is non-compact or the external scale factor is much greater than the internal one. First we discuss the case of the untwisted field with mass M satisfying the periodicity condition on S^1 . The vacuum energy density and the pressures along the uncompactified (p_0) and compactified (p_1) directions are given by the expressions

$$\begin{aligned}\rho &= -\frac{2M^{d_0+2}}{(2\pi)^{d_0/2+2}} \sum_{m=1}^{\infty} \frac{K_{d_0/2+1}(Mam)}{(Mam)^{d_0/2+1}}, \quad p_0 = -\rho, \\ p_1 &= -\rho - \frac{2M^{d_0+2}}{(2\pi)^{d_0/2+2}} \sum_{m=1}^{\infty} \frac{K_{d_0/2+2}(Mam)}{(Mam)^{d_0/2}},\end{aligned}\quad (13)$$

where $K_\nu(x)$ is the modified Bessel function of the second kind. The corresponding effective potential has the form

$$U = \frac{1}{2} \left(a^{(0)}/a \right)^{\frac{2}{d_0-1}} (16\pi G\rho + \Lambda_D), \quad (14)$$

with a constant $a^{(0)}$. The condition for the presence of the static internal space takes the form

$$d_0\rho - (d_0 - 1)p_1 = \frac{\Lambda_D}{8\pi G}. \quad (15)$$

From formulae (13) it follows that the left hand side of this expression is positive and, hence, the static solutions are present only for the positive cosmological constant Λ_D . The effective cosmological constant is equal $\Lambda_{\text{eff}} = -8\pi G(d_0 - 1)(\rho + p_1)$ and is positive. However, as it can be checked $(\partial/\partial a)[d_0\rho - (d_0 - 1)p_1] < 0$ and the solutions with static internal spaces are unstable.

In the case of a twisted scalar with antiperiodicity condition along the compactified dimension the corresponding energy density and pressures are obtained by using the expressions (13) with the help of the formula

$$q_t(a) = 2q(2a) - q(a), \quad q = \rho, p_0, p_1. \quad (16)$$

For the twisted scalar the corresponding energy density is positive. The solutions with static internal spaces are stable and the effective cosmological constant is negative: $\Lambda_{\text{eff}} < 0$.

In Figure 3 we have plotted the effective potential U in units of

$$U_0 = 4\pi G M^{d_0+2} (M a^{(0)})^{\frac{2}{d_0-1}}, \quad (17)$$

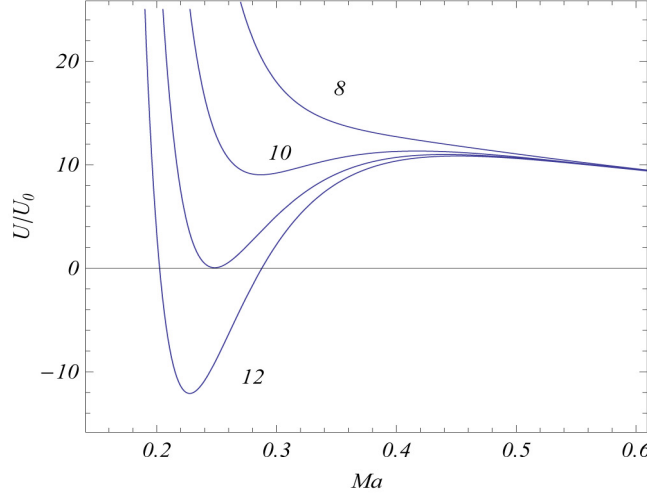


Figure 3: The potential U/U_0 in $d_0 = 3$ as a function of Ma for a model with single untwisted and two twisted scalars and for $\Lambda_D/(8\pi GM^{d_0+2}) = 2$. The numbers near the curves correspond to the values of the ratio M_t/M . For the graph with zero effective cosmological constant $M_t/M = 11.134$.

as a function on Ma for the model with $d_0 = 3$ consisting a single untwisted scalar with mass M and two twisted scalars with masses M_t for various values of the ratio M_t/M (numbers near the curves) and for $\Lambda_D/(8\pi GM^{d_0+2}) = 2$. For the graph with zero effective cosmological constant $M_t/M \approx 11.134$.

As we see in this type of models the size of the internal space is of the order $1/M$. The effective cosmological constant is of the order $\Lambda_{\text{eff}} \sim 8\pi M_{\text{Pl}}^{-2} \times 10^{-2}/a^4$. Setting the value of the corresponding energy density equal to the density of the dark energy, for the size of the internal space we find $a \sim 10^{-3}\text{cm}$. Such large extra dimensions are realized in models where the standard model fields are localized on a 4-dimensional hypersurface (brane). Note that the value of the effective cosmological constant can be tuned by the parameter M_t/M and the observed value of the dark energy density can also be obtained for smaller values for the size of the internal space. Similar results are obtained in models with more complicated internal spaces, in particular, in ADD models with the number of internal dimensions ≥ 2 .

Note that large extra dimensions accessible to all standard model fields can also be realized. This type of extra dimensions are referred to as universal dimensions. The key element in these models is the conservation of momentum in the Universal Extra Dimensions which leads to the Kaluza-Klein number conservation. In particular there are no tree-level contributions to the electroweak observables. The Kaluza-Klein modes may be produced only in groups of two or more and none of the known bounds on extra dimensions from single Kaluza-Klein production at colliders applies for universal extra dimensions. Contribution to precision electroweak observables arise at the one loop level. In the case of a single extra dimension, recent experimental constraints allow a compactification scale as low as TeV scale.

4 Brane models

Recently it has been suggested that the introduction of compactified extra spatial dimensions may provide a solution to the hierarchy problem between the gravitational and electroweak mass scales [31, 32]. The main idea to resolve the large hierarchy is that the small coupling of four-

dimensional gravity is generated by the large physical volume of extra dimensions. These theories provide a novel setting for discussing phenomenological and cosmological issues related to extra dimensions. The model introduced by Randall and Sundrum is particularly attractive. Their background solution consists of two parallel flat branes, one with positive tension and another with negative tension embedded in a five-dimensional AdS bulk [32]. The fifth coordinate is compactified on S^1/Z_2 , and the branes are on the two fixed points. It is assumed that all matter fields are confined on the branes and only the gravity propagates freely in the five-dimensional bulk. In this model, the hierarchy problem is solved if the distance between the branes is about 40 times the AdS radius and we live on the negative tension brane. More recently, scenarios with additional bulk fields have been considered.

For the braneworld scenario to be relevant, it is necessary to find a mechanism for generating a potential to stabilize the distance between the branes. The braneworld corresponds to a manifold with boundaries and all fields which propagate in the bulk will give Casimir-type contributions to the vacuum energy and, as a result, to the vacuum forces acting on the branes. Casimir forces provide a natural mechanism for stabilizing the radion field in the Randall-Sundrum model, as required for a complete solution of the hierarchy problem. In addition, the Casimir energy gives a contribution to both the brane and bulk cosmological constants and, hence, has to be taken into account in the self-consistent formulation of the braneworld dynamics.

In the $(D+1)$ -dimensional generalization of the Randall-Sundrum spacetime the background spacetime is described by the line-element

$$ds^2 = e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2, \quad (18)$$

where $\eta_{\mu\nu}$ is the metric tensor for the D -dimensional Minkowski spacetime the AdS curvature radius is given by $1/k$. The fifth dimension y is compactified on an orbifold, S^1/Z_2 of length a , with $-a < y < a$. The orbifold fixed points at $y = 0$ and $y = a$ are the locations of two D -branes. Consider a scalar field $\varphi(x)$ with curvature coupling parameter ζ obeying boundary conditions $(\tilde{A}_y + \partial_y) \varphi(x) = 0$, $y = 0, a$, on the branes. For a scalar field with brane mass terms c_0 and c_a on the left and right branes respectively, the coefficients in the boundary conditions are defined by the relation $2\tilde{A}_j = -n^{(j)}c_j - 4D\zeta k$ with $n^{(0)} = 1$, $n^{(a)} = -1$ (see, for instance, [18, 27, 33]). The corresponding Casimir energy is given by the expression [15, 16, 18, 27]

$$E(a) = \alpha + \beta e^{-Dka} + \frac{(4\pi)^{-D/2} k^D}{\Gamma(D/2)} \int_0^\infty du u^{D-1} \ln \left| 1 - \frac{\bar{I}_\nu^{(a)}(u) \bar{K}_\nu^{(b)}(ue^{ka})}{\bar{K}_\nu^{(a)}(u) \bar{I}_\nu^{(b)}(ue^{ka})} \right|, \quad (19)$$

where $I_\nu(u)$ and $K_\nu(u)$ are the modified Bessel functions,

$$\nu = \sqrt{(D/2)^2 - D(D+1)\zeta + m^2/k^2}, \quad (20)$$

and the barred notation for a given function $F(x)$ is defined by $\bar{F}^{(j)}(x) = (\tilde{A}_j/k + D/2)F(x) + xF'(x)$. The first and second terms on the right of Eq. (19) are the energies for a single brane located at $y = 0$ and $y = a$ respectively when the second brane is absent. The coefficients α and β cannot be determined from the low-energy effective theory and can be fixed by imposing suitable renormalization conditions which relate them to observables. The last term in (19) is free of renormalization ambiguities and can be termed as an interaction part. The corresponding vacuum forces acting on the branes can be either repulsive or attractive in dependence of the coefficients in the boundary conditions. In particular, there is a region in the parameter space where these forces are attractive at large distances between the branes and repulsive at small distances with an equilibrium state at some intermediate distance $a = a_0$. In the original Randall-Sundrum braneworld with $D = 4$, to account for the observed hierarchy between the gravitational and electroweak scales we need $ka_0 \approx 37$.

In addition to the stabilization of the distance between the branes, the quantum effects from bulk fields can also provide a mechanism for the generation of the cosmological constant on the visible brane. On manifolds with boundaries in addition to volume part, the vacuum energy contains a contribution located on the boundary. For a scalar field the surface energy density $\varepsilon_v^{(\text{surf})}$ on the visible brane $y = a$ is presented as the sum [34]

$$\varepsilon_v^{(\text{surf})} = \varepsilon_{1v}^{(\text{surf})} + \Delta\varepsilon_v^{(\text{surf})}, \quad (21)$$

where $\varepsilon_{1v}^{(\text{surf})}$ is the surface energy density on the visible brane when the hidden brane is absent, and the part

$$\Delta\varepsilon_v^{(\text{surf})} = k^D \frac{(4\zeta - 1)\tilde{A}_j - 2\zeta}{(4\pi)^{D/2}\Gamma(D/2)} \int_0^\infty du \frac{u^{D-1} \bar{I}_\nu^{(a)}(ue^{-ka})/\bar{I}_\nu^{(b)}(u)}{\bar{K}_\nu^{(a)}(ue^{-ka})\bar{I}_\nu^{(b)}(u) - \bar{I}_\nu^{(a)}(ue^{-ka})\bar{K}_\nu^{(b)}(u)}, \quad (22)$$

is induced by the presence of the hidden brane. The part $\varepsilon_{1v}^{(\text{surf})}$ contains finite renormalization terms which are not computable within the framework of the model under consideration and their values should be fixed by additional renormalization conditions. The effective cosmological constant generated by the hidden brane is determined by the relation

$$\Lambda_v = 8\pi M_v^{2-D} \Delta\varepsilon_v^{(\text{surf})}, \quad (23)$$

where M_v is the D -dimensional effective Planck mass scale for an observer on the visible brane. For large interbrane distances the quantity (23) is suppressed compared with the corresponding Planck scale quantity in the brane universe by the factor $\exp[-k(2\nu + D)a]$, assuming that the AdS inverse radius and the fundamental Planck mass are of the same order. In the original Randall-Sundrum model with $D = 4$, for a scalar field with the mass $|m|^2 \lesssim k^2$, and interbrane distances solving the hierarchy problem, the value of the induced cosmological constant on the visible brane by the order of magnitude is in agreement with the value of the dark energy density suggested by current cosmological observations without an additional fine tuning of the parameters.

5 Exotic Propulsion

The idea of manipulating spacetime in some ingenious fashion to facilitate a novel form of spacecraft propulsion has been well explored in the literature [35–55]. If we are to realistically entertain the notion of interstellar exploration in timeframes of a human life-span, a profound shift in the traditional approach to spacecraft propulsion is clearly necessary. It is well known that the universe imposes dramatic relativistic limitations on all bodies moving through spacetime, and that all matter is restricted to motion at sublight velocities ($< 3 \times 10^8 \text{m/s}$, the speed of light, or c) and that as matter approaches the speed of light, its mass asymptotically approaches infinity. This mass increase ensures that an infinite amount of energy would be necessary to travel at the speed of light, and thus, this speed is impossible to reach and represents an absolute speed limit to all matter travelling through spacetime.

Even if an engine were designed that could propel a spacecraft to an appreciable fraction of light-speed, travel to even the closest stars would take many decades in the frame of reference of an observer located on Earth. Although these lengthy transit times would not make interstellar exploration impossible, they would certainly reduce the enthusiasm of governments or private individuals funding these missions. After all, a mission whose success is, perhaps, a century away would be difficult to justify. In recent years, however, two loop-holes to Einstein's ultimate speed

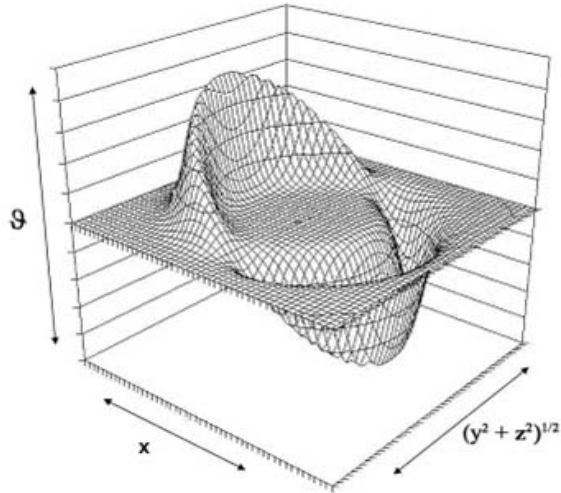


Figure 4: The ‘Top hat metric’. A bubble of asymmetric spacetime curvature surrounds a spacecraft which would sit in the center of the bubble. The space immediately surrounding the spacecraft would be expanding/contracting behind/in front of the craft. In this image the ship would ‘move’ from left to right.

limit are known to exist: the Einstein-Rosen bridge and the warp-drive. Fundamentally, both ideas involve the manipulation of spacetime itself in some exotic way that allows for superluminal travel.

The warp drive, which is the main feature of study for this section, involves local manipulation of the fabric of space in the immediate vicinity of a spacecraft. The basic idea is to create an asymmetric bubble of space that is contracting in front of the spacecraft while expanding behind it.² Spacetime is believed to have stretched at many times the speed of light in the first second of its existence during the inflationary period. In many ways, the idea presented in this paper is an artificial and local re-creation of those initial conditions.

Using this form of locomotion, the spacecraft remains stationary inside this ‘warp bubble,’ and it is the movement of space itself that facilitates the relative motion of the spacecraft. The most attractive feature of the warp drive is that the theory of relativity places no known restrictions on the motion of space itself, thus allowing for a convenient circumvention of the speed of light barrier.

By associating dark energy with the Casimir energy due to the KK modes of vacuum fluctuations in higher dimensions, especially in the context of M-theory derived or inspired models, it is possible to form a relationship between Λ and the radius of the compact extra dimension.

$$\rho = \Lambda \propto 1/a^D. \quad (24)$$

An easier way of developing the relationship between the energy density and the expansion of space is to discuss quantities in terms of Hubble’s constant H , which describes the rate of expansion of space per unit distance of space.

$$H \propto \sqrt{\Lambda}, \quad (25)$$

²Recent progress by José Natário has demonstrated that, with a slightly more complicated metric, one can dispense of the expansion.

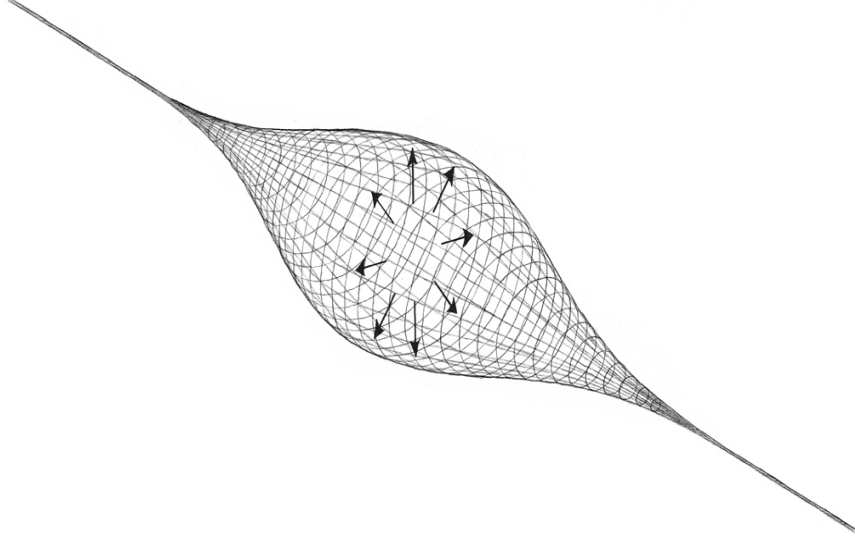


Figure 5: An extra dimension that is artificially stimulated to contract (or expand), due to interaction with the SM fields, might provide a way to locally adjust the dark energy density.

or in terms of the radius of the extra dimension we have

$$H \propto 1/a^{D/2}. \quad (26)$$

This result indicates that within this model, the expansion of spacetime is a function of the size of the higher dimension. One fascinating question to ask at this point is: could it be possible to effect the radius of a higher dimension through some advanced technology? If this were, indeed, possible then this technology would provide a remarkable mechanism to locally adjust the dark energy density, and thus the local expansion rate of spacetime (Figure 5).³

In principal, this represents an interesting way to artificially generate the type of spacetime featured in Figure 4. A spacecraft with the ability to create such a bubble would always move inside its own local light-cone. This ship could utilize the expansion of spacetime behind the ship to move away from some object at any desired speed, or equivalently, to contract the space-time in front of the ship to approach any object. The possibility that the size of the compact geometry might, indeed, vary depending on the location in four dimensional spacetime has been explored in the context of string theory [56], but never from the perspective of propulsion technology.

To explore the types of energies that may be required to generate this manipulation of a higher dimension, we consider that fact that in models with large extra dimensions, the interaction of the graviton KK tower with the Standard Model fields are suppressed by the higher dimensional Planck scale, and the corresponding couplings are inverse TeV in strength. This can be seen more clearly when we consider the expansion of the metric tensor in models with large extra dimensions computed within linearized gravity models:

$$g_{MN} = \eta_{MN} + h_{MN}/M_D^{D/2-1}, \quad (27)$$

where η_{MN} corresponds to flat (Minkowski) spacetime and h_{MN} corresponds to the bulk graviton fluctuations. The graviton interaction term in the action is expressed by:

$$S_{\text{int}} = 1/M_D^{D/2-1} \int d^D x h_{MN} T^{MN} \quad (28)$$

³Illustration courtesy of Richard Obousy Consulting LLC and AlVin, Antigravit 

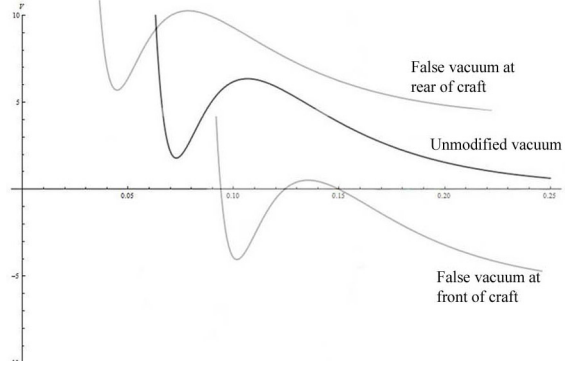


Figure 6: By surrounding a spacecraft with false minima an artificial inflation of the local spacetime might be achieved.

with T^{MN} being the higher dimensional energy-momentum tensor. The interaction of the graviton KK states with the SM fields are obtained by integrating the action over the extra coordinates. Because all these states are coupled with the universal strength $1/M_{\text{Pl}}$ this leads to the compelling possibility of the control of the size of the extra dimensions by processes at energies that will be accessible via the particle accelerators of the near future. Although the coupling is extremely small, the effective coupling is enhanced by the large number of KK states.

Referring to 2, additional energy in the form of matter or radiation with the TeV energy scale can alter the shape of the effective potential. In particular, the extrema determining the size of the extra dimensions are modified with the change of the Casimir energy density and hence, the dark energy density, in the models under consideration.

The key to creating a warp drive in this model is to create a false vacuum minimum, i.e., to modify the vacuum spectrum and inject some field which creates a deSitter minimum at the rear of the craft and an anti-deSitter minimum at the front of the craft. What this requires is a technology that would allow us to artificially manipulate the field content illustrated in 2, shifting the location of the minimum. In this basic representation, the spacecraft would sit in a stable region of space corresponding to the natural minimum of the extra dimension (approximately flat space). At the front and rear of the craft, regions of false minima would be artificially created via the adjustment of the extra dimension (Figure 6). These modified regions would correspond to increased and negative dark energy densities, thus creating the warp bubble previously discussed.

6 Extra Dimensions in General Relativity

In the context of GR a similar phenomenology is produced for the case of anisotropic cosmological models, in which it is the *contraction* of the extra dimension that has the effect of expanding another [57]. For example, consider a ‘toy’ universe with one additional spatial dimension with the following metric

$$ds^2 = dt^2 - a^2(t)d\vec{x}^2 - b^2(t)dy^2 . \quad (29)$$

In this toy universe we will assume spacetime is empty, that there is no cosmological constant, and that all spatial dimensions are locally flat,

$$T_{\mu\nu} = \Lambda g_{\mu\nu} = 0 . \quad (30)$$

The action of the Einstein theory of gravity generalized to five dimensions will be

$$S^{(5)} = \int d^4x dy \sqrt{-g^{(5)}} \left(\frac{M_5^2}{16\pi} R^{(5)} \right) . \quad (31)$$

Solving the vacuum Einstein equations

$$G_{\mu\nu} = 0 , \quad (32)$$

we obtain for the G_{11} component

$$G_{11} = \frac{3\dot{a}(b\dot{a} + a\dot{b})}{a^2b} . \quad (33)$$

Rewriting $\dot{a}/a = H_a$ and $\dot{b}/b = H_b$ where H_a and H_b corresponds to the Hubble constant in three space and the Hubble constant in the extra dimension respectively, we find that solving for $G_{11} = 0$ yields

$$H_a = -H_b. \quad (34)$$

This remarkable result indicates that in a vacuum, the shear of a contracting dimension is able to inflate the remaining dimensions. In other words, the expansion of the 3-volume is associated with the contraction of the one-volume.

Even in the limit of flat spacetime with zero cosmological constant, general relativity shows that the physics of the *compactified* space affects the expansion rate of the non-compact space. The main difference to note here is that the quantum field theoretic result demonstrates that a *fixed* compactification radius can also result in expansion of the three-volume due to the Casimir effect, whereas the GR approach suggests that a *changing* compactification radius results in expansion, as is shown in (34). What is particularly interesting about this result is that it demonstrates that the expansion and contraction of the non-compact space seems to be intimately related to the extra dimension in both QFT and GR calculations.

7 Discussion

As we have seen, the stabilization of compact extra dimensions and the acceleration of the other 4-dimensional part of the spacetime can be simultaneously described by using the Casimir energy-momentum tensor as a source in the Einstein equations. The acceleration in the 3-dimensional subspace occurs naturally when the extra dimensions are stabilized. In this case the Casimir energy density ρ_C is a constant with the equation of state $p_C = -\rho_C$, where p_C is the Casimir pressure in the visible universe, and an effective cosmological constant is induced. Note that the current observational bounds for the equation of state parameter are $-1.4 < p_{DE}/\rho_{DE} < -0.85$ and the value -1 of this parameter is among the best fits of the recent observational data. For the Casimir energy density to be the dark energy driving accelerated expansion the size of the internal space should be of the order 10^{-3}cm . Such large extra dimensions are realized in braneworld scenario. The value of the effective cosmological constant can be tuned by choosing the masses of the fields and the observed value of the dark energy density can also be obtained for smaller values for the size of the internal space.

An important feature of both models with a smooth distribution of matter in the extra dimensions and with branes is the dependence of the dark energy density on the size of the extra dimensions. In models with large extra dimensions the interaction of the graviton Kaluza-Klein tower with the standard model fields are suppressed by the higher dimensional Planck scale and the corresponding couplings are inverse TeV strength. This leads to the interesting

possibility for the control of the size of extra dimensions by processes at energies accessible at the near future particle accelerators. This is seen from the form of the effective potential for the scale factor of the internal subspace given above. Additional energy density in the form of matter or radiation with the TeV energy scale can alter the shape of the effective potential. In particular, the extrema determining the size of the extra dimensions are changed with the change of the Casimir energy density and, hence, the dark energy density in the models under consideration.

In the ADD scenario the Standard model fields are confined to a 4D brane and a new gravity scale $M_D = G^{1/(2-D)} \gtrsim \text{TeV}$ is introduced in $4 + d_1$ dimensions. The gravity on the brane appears as a tower of Kaluza-Klein states with universal coupling to the Standard Model fields. Though this coupling is small ($\sim 1/M_{\text{Pl}}$), a relatively large cross section is obtained from the large number of Kaluza-Klein states. Kaluza-Klein graviton emission from SN1987A puts a bound $M_D \geq 30 \text{ TeV}$ on the fundamental energy scale for ADD type models in the case $d_1 = 2$. For the models with $d_1 = 3$ the constraint is less restrictive, $M_D \geq \text{few TeVs}$. Hence, the latter case is still viable for solving the hierarchy problem and accessible to being tested at the LHC. The ATLAS experiment which will start taking data at the LHC will be able to probe ADD type extra dimensions up to $M_D \approx 8 \text{ TeV}$. In the Randall-Sundrum scenario the hierarchy is explained by an exponential warp factor in AdS_5 bulk geometry. Electroweak precision tests put a severe lower bound on the lowest Kaluza-Klein mass. The masses of the order of 1 TeV can be accommodated. In universal extra dimension scenarios the current Tevatron results constrain the mass of the compactification scale to $M_C > 400 \text{ GeV}$. The ATLAS experiment will be sensitive to $M_C \approx 3 \text{ TeV}$. These estimates show that if the nature is described by one of the models under consideration, the extra dimensions can be probed and their size can be controlled at the energies accessible at LHC. In models with the Casimir energy in the role of the dark energy this provides a possibility for the control of the dark energy local density through the change in the size of the extra dimensions.

Note that the effective potential (5) contains a factor $\Omega^{-2} \sim V_{\text{int}}^{-2/(d_0-1)}$, with V_{int} being the volume of the internal space, and, hence, vanishes in the large volume limit for the extra dimensions if the energy density grows not sufficiently fast. This leads to the instability of a dS minimum with respect to the decompactification of extra dimensions. This feature is characteristic for other stabilization mechanisms as well including those based on fluxes from form-fields and wrapped branes. The stability properties of the metastable dS minimum depend on the height of the local maximum separating this minimum from the minimum which corresponds to the infinite size of the extra dimensions. Now we conclude that in models with large extra dimensions, as in the case of the parameters for the minimum, the height and the location of the maximum can be controlled by the processes with the energy density of the TeV scale.

Another important point which should be touched is the following. Though the size of the internal space is stabilized by the one-loop effective potential, in general, there are fluctuations around the corresponding minimum with both classical and quantum sources which shift the size for the internal space from the fixed value leading to the time variations of this size. This gives time variations in the equation of state parameter for the corresponding dark energy density. However, it should be taken into account that the variations of the size for the internal space around the minimum induces variation of fundamental constants (in particular, gauge couplings) in the effective four-dimensional theory. These variations are strongly constrained by the observational data. As a consequence, the variation of the equation of state parameter for the corresponding dark energy density around the mean value -1 are small. It is interesting that in the models under consideration the constancy of the dark energy density is related to the constancy of effective physical constants. Note that models can be constructed where the volume of the extra dimensions is stabilized at early times, thus guaranteeing standard four

dimensional gravity, while the role of quintessence is played by the moduli fields controlling the shape of the extra space.

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